Powers of Ten

A book about the relative size of things in the universe
and the effect of adding another zero

Philip and Phylis Morrison
and
The Office of Charles and Ray Eames
based on the film *Powers of Ten*
by The Office of Charles and Ray Eames

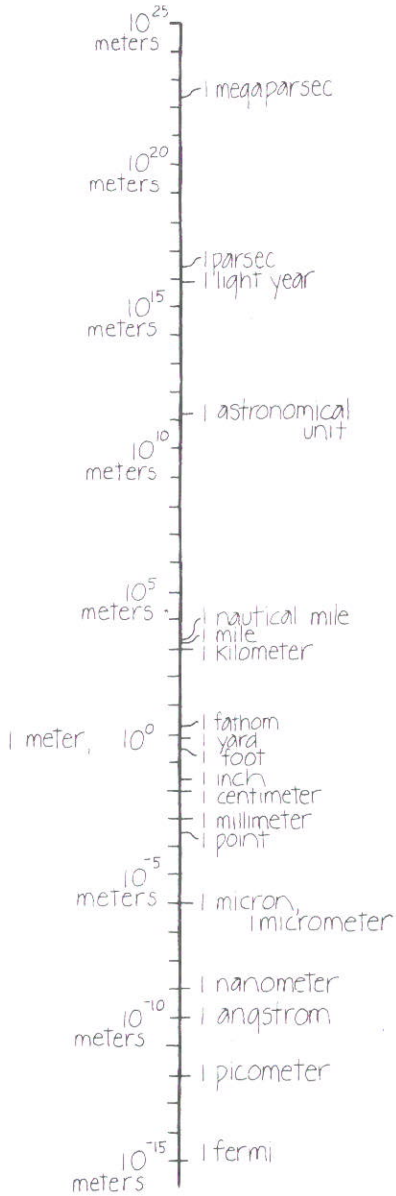
POWERS OF 10: HOW TO WRITE NUMBERS LARGE AND SMALL

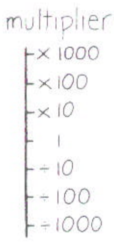
This book uses a notation based on counting how many times 10 must be multiplied by itself to reach an intended number: For example, 10×10 equals 10^2, or 100; and $10 \times 10 \times 10$ equals 10^3, or 1000. Multiplying a number by itself produces a *power* of that number: 10^3 is read out loud as "ten to the third power," and is another way to say one thousand. In this case, there is no great advantage, but it is much easier and clearer to write or say 10^{14} than 100,000,000,000,000 or one hundred trillion. After 10^{14}, we even run low on names. The number written above in smaller type—the 14 in the last example—is called an *exponent,* and the powers notation is often called *exponential notation.*

It is not hard to grasp the positive powers of ten—10^4, 10^7, 10^{19}—and how they work; but the negative powers—10^{-2} or 10^{-3}—are another matter. If the exponent tells how many times the 10 is to be self-multiplied, what can an exponent of -5 (negative five) mean? The system requires a negative exponent to signal division by 10 a certain number of times: 10^{-1} equals 1 divided by 10, or 0.1 (one-tenth); 10^{-2} equals 0.1 divided by 10, or 0.01 (one-hundredth). Because *adding* 1 to the exponent easily works out to be the equivalent of multiplying by 10, it is self-consistent that subtracting 1 there works out to a division by 10. It is all a matter of placing zeros. Adding another terminal zero is simply to multiply by 10: 100×10 equals 1,000. Putting another zero next after the decimal point is to *divide* by 10: $0.01 \div 10$ equals 0.001. The powers notation makes these operations even clearer.

But what of 10^0? That seems a strange number. However, notice it is equal to 10^1 (10) divided by 10 (or to 10^{-1} multiplied by 10). Although surprising, it is at least logical that 10^0 should be equal to 1.

Because you can make any power of ten ten times larger by adding 1 to its exponent ($10^4 \times 10 = 10^5$), it follows that to mul-





tiply by 100 you add 2 to the exponent: $10^3 \times 100 = 10^5$ or $1000 \times 100 = 100{,}000$. In general, you can multiply one power of ten by another simply by adding their exponents: $10^6 \times 10^3 = 10^9$. Subtracting the exponents is the equivalent of division: $10^7 \div 10^5 = 10^2$.

All numbers, not only numbers that are exact powers of ten, like 100 or 10,000, can be written with the help of exponential notation. The number 4000 is 4×10^3; 186,000 is 1.86×10^5. This convenient scheme is referred to as scientific notation.

All of this can be extended to basic multipliers other than ten: $2^4 = 2 \times 2 \times 2 \times 2$ (the fourth power of two); $12^2 = 12 \times 12$, and 8^{-1} = one-eighth. (But note that $2^0 = 1$, $12^0 = 1$, and $8^0 = 1$.)

Logarithms arise from extensions of this scheme.

The symbol ~ is mathematicians' shorthand for "approximately" or "about."

UNITS OF LENGTH

Grow you own food and build your own house, and no formal units of measurement much interest you; such is the general rule of thumb. But commerce has implied agreement on units of measurement. The legal yard has long been displayed for the use of Londoners, and the meter is still open to public comparison on a wall of a Paris building.

The system we call metric is the work of the savants of Revolutionary Paris in the 1790s. Even their determination to celebrate both novelty and reason met limits: Our modern second, minute, and hour remain resolutely nondecimal. That was no oversight—the metric day of ten hours, each of a hundred minutes with a hundred seconds to the minute, was formally adopted. But the scheme met fierce resistance. About the only costly mechanism every middle-class family then proudly owned was a clock or watch, not to be rendered at once useless by any mere claim of consistency! Practice won out over theory.

In much the same way, people who today frequently use units in a particular context are not always persuaded to sacrifice appropriateness to consistency. We list here a few nonstandard units of linear measurement that retain their utility even in these metric days, some even within the sciences.

Cosmic Distances

Parsec The word is a coinage from *par*allax of one *sec*ond. The *parsec* is in common use among astronomers because it hints at the surveyor's basic technique of measuring stellar distance by using triangulation. The standard parallax is the apparent shift in direction of a distant object at six-month intervals as the observer moves with the orbiting earth. It is defined so that the radius of the earth's orbit seen from a distance of one parsec spans an angle of one second of arc. The nearest known star to the sun is more than one parsec away.

Light-year This graspable interstellar unit rests on the relationship between cosmic distance and light travel over time.

The speed of light in space is 3.00×10^8 meters per second; in one year light thus moves 9.46×10^{15} meters, which is usually rounded off to 10^{16} meters, especially since only a few cosmic distances are so well known that the roundoff is any real loss of accuracy.

Astronomical unit The mean distance between sun and earth is a fine baseline for surveying the solar system; it is a typical length among orbits. 1 AU = 1.50×10^{11} meters. *Note:* 1 parsec = 3.26 light-years = 206,300 AU. The interstellar and the solar-system scales plainly differ; intergalactic distances run to megaparsecs.

Terrestrial Lengths

Miles, leagues, etc. These are units suited for earthbound travel, for distances at sea, or for road distances between cities. Nobody ever measured cloth by the mile, or train rides by the parsec.

Yards, feet, meters These rest on human scale, in folklore the length of some good king's arm. They suit well the sizes of rooms, people, trucks, boats. Textiles are yard goods. The meter was defined more universally, but clearly it was meant to supplant the yard and the foot. It was related to the size of the earth: One quadrant of the earth's circumference was defined as exactly 10^7 meters, or 10^4 kilometers. In 1981 the meter is defined with great precision in terms of the wavelength of a specific atomic spectral line. It is "1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transitions between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom."

Inches, centimeters, etc. The same king's thumb? Human-scale units intended for the smaller artifacts of the hand: paper sizes, furniture, hats, or pies.

Line, millimeter, point Small units for fine work are relatively modern. The seventeenth-century French and English line was a couple of millimeters, and the printer's point measure is about 0.35 mm. Film, watches, and the like are commonly sized by millimeters. The pioneer microscopist Antony van Leeuwenhoek used sand grains as his length comparison, coarse and fine: He counted one hundred of the fine grains to the common inch of his place and time. Smaller measurement units are generally part of modern science, and thus usually metric.

Atomic Distances

Angstroms, fermis, etc. Once atoms became the topic of meaningful measurement, new small units of length naturally came into specialized use. The Swedish physicist Anders Ångström a century ago pioneered wavelength measurements of the solar spectrum. He expressed his results in terms of a length unit just 10^{-10} meters long. It has remained in widespread informal use bearing his name: convenient because atoms measure a few angstroms (Å) across. The impulse for such useful jargon words is by no means ended; nuclear particles are often measured in fermis, after the Italian physicist Enrico Fermi. 1 fermi equals 10^{-15} meters.

Angles and Time

Angles are measured, especially in astronomy, by a nonmetric system that goes back to Babylon. A circle is 360 degrees; 1 degree = 60 minutes of arc; 1 minute = 60 arc-seconds. An arc-second is roughly the smallest angle that the image of a star occupies, smeared as it is by atmospheric motion. This page, viewed from about twenty-five miles away, would appear about one arc-second across.

Time measurement shares the cuneiform usage of powers of sixty. Note that a year of 365.25 days of 24 hours, each of 60 minutes with 60 seconds apiece, amounts to about 3.16×10^7 seconds.